**Lecture** **5.**

**One-Sided Limits. Infinite limits. Monotonic Functions.**

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 We write **** to indicate that as $x$ approaches $a$ from the right, $f(x)$ approaches $b$, i.e. if $x>a and x\rightarrow a$ , then we write conventionally $x\rightarrow a+0.$ The number  is called the *limit from the right* of the function.

**Definition . (**theleft-hand limit of $f(x)$ ).

 We write **** to indicate that as $x$ approaches $ a$ from the left, $f(x)$ approaches $b,$ i.e. if $x<a and x\rightarrow a$ , then we write conventionally $x\rightarrow a-0.$ The number  is called the *limit from the left* of the function.

**Definition.** The right-hand limit and left-hand limit are called *one-sided limits.*

*For a full limit to exist, both one-sided limits have to exist and they have to be equal.*

 **First remarkable limit :**

  (4)

The formula (4) is frequently used when solving the following examples.

**Example 1:** 

**Example 2:**  

**Theorem (**The squeezing theorem). If *f, g*, and *h* be functions satisfying

$$h\left(x\right)\leq f\left(x\right)\leq g\left(x\right).$$

Let *p>0*. Suppose that, for all *x* such that

$$0<\left|x-c\right|<p.$$

If $\lim\_{x\to c}h\left(x\right)=L and \lim\_{x\to c}g\left(x\right)=L $, then

$$\lim\_{x\to c}f\left(x\right)=L .$$

**Monotonic Functions**

**Definition.** The function *f* is increasing on an interval I, if $f\left(x\_{1}\right)<f(x\_{2})$ whenever $x\_{1} and x\_{2} $are in I, and $x\_{1}<x\_{2}$, or decreasing on an interval I, if $f\left(x\_{1}\right)>f(x\_{2})$ whenever $x\_{1} and x\_{2} $are in I, and $x\_{1}>x\_{2}.$

In either of these two case, f is strictly monotonic on I.

The function *f* is nondecreasing on an interval I, if $f\left(x\_{1}\right)\leq f(x\_{2})$ whenever $x\_{1} and x\_{2} $are in I, and $x\_{1}<x\_{2}$, or nonincreasing on an interval I, if $f\left(x\_{1}\right)\geq f(x\_{2})$ whenever $x\_{1} and x\_{2} $are in I, and $x\_{1}>x\_{2}.$

A functions that satisfies any of these conditions is called *monotonic*.

**Definition.**

We say that *f* is bounded on a set S, if there is a constant $M<\infty $ such that

$$\left|f(x)\right|<M$$

for all x in S.